

# Bistatic Radar Denial/Embedded Communications Via Waveform Diversity

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**Abstract** – Use of an interferometer along with a host radar is proposed for simultaneously achieving coherent reference denial and embedded communications. To prevent self jamming, spatial orthogonality is achieved between the interferometer antenna pattern and main beam of the host radar. Costas and orthogonal frequency division multiplexing (OFDM) signals are suggested for the host radar and interferometer, respectively. Effectiveness of the interferometer masking signal on a non-cooperative bistatic radar is discussed.

## I. INTRODUCTION

Of increasing concern is the threat provided by relatively inexpensive noncooperative radars which operate bistatically by hosting off of illuminations from advanced surveillance systems. A central requirement for noncooperative bistatic operation is estimation of a coherent reference signal. This estimate is used to correlate against received waveforms to detect the desired signal. A coherent reference is typically obtained by measuring a direct path signal emitted through the sidelobes of the host radar. Conventional methods to prevent interception of the direct path signal include low sidelobe antennas, physical isolation, and the use of spread spectrum waveforms. These methods are expected to become inadequate as sensor technology advances. In addition, it is desirable to incorporate embedded communications with the host radar surveillance function in order to enhance precision engagement.

This paper discusses an approach involving the employment of an  $(N+n)$ -element linear antenna array where  $N$  of the elements are used by the host radar for surveillance while  $n$  of the elements are used by the interferometer for both coherent reference denial and communications. The  $n$  interferometric elements are driven separately using independent waveform generation, timing and control circuits.

The interferometer signal is intended to be 1) effective in masking the host radar signal emitted through the sidelobes, 2) orthogonal to the host radar signal transmitted through the radar main beam so as not to degrade radar performance, 3) useful for data/voice communication and 4) steerable such that separate communication signals can be directed to multiple receivers located in different directions. Also, the host radar signal should be flexible such that it is suitable for multi-mission operations.

## II. ORTHOGONALITY CONSIDERATIONS

To avoid self jamming, the interferometer and radar signals should be mutually orthogonal. Assume a target is illuminated by both the interferometer and main beam of the host radar. The target return will then consist of a weighted sum of the two signals. Let  $s_R(t)$  and  $s_I(t)$  denote the complex envelopes of the radar and interferometer signals, respectively. The complex envelope of the signal received by the host radar can be expressed on

$$r_{HR}(t) = k_1 s_R(t) + k_2 s_I(t) \quad (1)$$

where  $k_1$  and  $k_2$  are constants that account for the host radar and interferometer antenna patterns in the target direction along with propagation and reflection losses. Assume the host radar detection statistic,  $\ell_{HR}$ , is obtained by correlating the complex envelopes of the received and transmitted radar signals. Assuming perfect synchronization, this is given by

$$\begin{aligned} \ell_{HR} &= \int_{-\infty}^{\infty} s_R^*(t) r_{HR}(t) dt \\ &= k_1 \int_{-\infty}^{\infty} |s_R(t)|^2 dt + k_2 \int_{-\infty}^{\infty} s_R^*(t) s_I(t) dt \end{aligned} \quad (2)$$

To avoid degradation by the interferometer, the term in  $\ell_{HR}$  involving  $s_i(t)$  should be zero. This can be accomplished either through temporal orthogonality such that

$$\int_{-\infty}^{\infty} s_R^*(t) s_i(t) dt = 0 \quad (3)$$

or through spatial orthogonality such that

$$k_2 = 0 \quad (4)$$

However, now focus on the detection statistic computed by the noncooperative radar. Assume that (4) is valid such that the target is illuminated only by the host radar. Consequently, the target return at the noncooperative radar is

$$r_{NR}(t) = k_3 s_R(t) \quad (5)$$

where  $k_3$  is a constant that accounts for the host and noncooperative radar antenna patterns in the path directions connecting the host radar to the target and the target to the noncooperative radar along with propagation and reflection losses. Assume the noncooperative radar uses as its estimate of the transmitted radar signal the direct path signal which is a weighted sum of the host radar and interferometer signals. This estimate is represented by

$$\hat{s}_R(t) = k_4 s_R(t) + k_5 s_i(t) \quad (6)$$

where  $k_4$  and  $k_5$  are constants accounting for antenna gains and losses along the direct path. Let the detection statistic,  $\ell_{NR}$ , be obtained by correlating the received and estimated signals. Hence,

$$\begin{aligned} \ell_{NR} &= \int_{-\infty}^{\infty} (\hat{s}_R(t))^* r_{NR}(t) dt \\ &= k_4^* k_3 \int_{-\infty}^{\infty} |s_R(t)|^2 dt + k_5^* \int_{-\infty}^{\infty} s_i^*(t) s_R(t) dt. \end{aligned} \quad (7)$$

Note that (7) is unaffected by the presence of the interferometer signal should (3) hold. Therefore, the radar and interferometer signals should not be temporally orthogonal. In fact, for effective masking, the interferometer signal should be highly correlated with the radar signal. It is concluded that spatial orthogonality is the

mechanism to be used to avoid self jamming by the interferometer.

### III. INTERFEROMETER ANTENNA PATTERN

Interferometers composed of  $n$ -element linear arrays are considered in this paper. The associated antenna array factor is denoted by  $F(n; \theta, \alpha)$  where  $n$  denotes the number of antenna elements,  $\theta$  is the elevation angle measured from the  $z$ -axis and  $\alpha$  is the azimuth angle measured from the  $y$ -axis.

The 2-element array is considered first. Assume the two antenna elements are placed along the  $x$ -axis at  $x = \pm d/2$ . The antenna array factor can be shown to be

$$F(2; \theta, \alpha) = \cos \left[ \pi \left( \frac{d}{\lambda} \right) \sin \theta \sin \alpha \right] \quad (8)$$

where  $\lambda$  is the wavelength of the sinusoidal excitation applied to both array elements. Let the element spacing be given by

$$d = k_s \frac{\lambda}{2} \quad (9)$$

where  $k_s$  is a positive integer. In the horizontal  $x$ - $y$  plane, the array factor becomes

$$F(2; \frac{\pi}{2}, \alpha) = \cos \left( k_s \frac{\pi}{2} \sin \alpha \right) \quad (10)$$

This array factor displays the following properties:

1. Independent of whether  $k_s$  is an odd or even integer, (a) symmetry exists about the  $x$ - and  $y$ -axes, (b) the number of major lobes equals  $2k_s$ , (c) there are no minor lobes, (d) major lobes always occur for  $\alpha = 0^\circ, 180^\circ$ , (e) the number of nulls equals  $2k_s$ .
2. When  $k_s$  is an even integer, (a) the polarity of the major lobes strictly alternate in sign from one major lobe to the next, (b) major lobes always occur for  $\alpha = \pm 90^\circ$ .
3. When  $k_s$  is an odd integer, (a) nulls always occur for  $\alpha = \pm 90^\circ$  and point in the end-fire positions along the  $x$ -axis, (b) the first lobes to either side of the  $x$ -axis have the same polarity, (c) the

polarity of the remaining major lobes strictly alternate in sign from one major lobe to the next.

It is important to distinguish between nulls which appear between successive major lobes of the same polarity versus those which appear between successive major lobes of opposite polarity. The former are broad nulls because the slope of the array factor is necessarily zero at these nulls since the array factor does not change sign between the peaks on either side of the nulls. On the other hand, the latter are narrow nulls because the slope of the array factor is nonzero at these nulls as the polarity changes between the neighboring peaks.

With the above observations, it is concluded that the array factor does not have any broad nulls when  $k_s$  is an even integer and has broad nulls only in the end-fire positions along the x-axis when  $k_s$  is an odd integer. For example, define the width of a null to be its angular extent between the  $-30$  dB points. When  $k_s = 3$ , the four narrow nulls have widths of approximately  $0.8^\circ$  while the two broad nulls in the end-fire position at  $\alpha = \pm 90^\circ$  have widths approximately equal to  $12.8^\circ$ . Thus, the broad nulls are approximately 16 times wider than the narrow nulls.

To avoid self jamming, it would be desirable to steer the interferometer pattern such that one of its broad nulls is centered in the main beam of the host radar. In this way, spatial orthogonality between the interferometer and host radar signals would be achieved. When a phase shift,  $\delta$ , is introduced between the two interferometer elements, the array factor is given by

$$F(2; \frac{\pi}{2}, \alpha, \delta) = \cos \left[ k_s \frac{\pi}{2} \sin \alpha + \frac{\delta}{2} \right]. \quad (11)$$

A null is steered to  $\alpha_0$  by requiring that

$$k_s \frac{\pi}{2} \sin \alpha_0 + \frac{\delta}{2} = p \frac{\pi}{2} \quad (12)$$

where  $p$  is an odd integer. For this to be a broad null, it is necessary that

$$\frac{dF(2; \frac{\pi}{2}, \alpha_0, \delta)}{d\alpha} = -\frac{\pi}{2} k_s \sin \left[ p \frac{\pi}{2} \right] \cos \alpha_0 = 0.$$

However, the derivative is zero only for (13)

$$\alpha_0 = \pm 90^\circ \quad (14)$$

which, of course, corresponds to the end-fire positions. It is concluded that the broad null cannot be steered.

Assume the antenna array elements of the host radar are placed along the x-axis and centered at the origin. The interferometer broad null can be positioned broad side to this array (i.e., at  $\alpha=0^\circ$ ) by placing the two interferometer elements along the y-axis. In addition, the broad null could be steered by constructing the interferometer from a single element placed along the y-axis and one of several elements placed along the x-axis such that the constraint in (9) is maintained. The interferometer broad null would then be steered by utilizing an appropriately placed element on the x-axis along with the single element on the y-axis.

An interferometer composed of  $n$  elements placed along the x-axis and equally spaced by an amount,  $d$ , is now considered. Assume the element excitations are weighted by the binomial coefficients which arise in the binomial expansion of  $(a+b)^n$ . The antenna array factor is given by

$$F(n; \theta, \alpha) = \left\{ \cos \left[ \pi \left( \frac{d}{\lambda} \right) \sin \theta \sin \alpha \right] \right\}^{n-1} \quad (15)$$

$$= \{F(2; \theta, \alpha)\}^{n-1}$$

where, once again,  $\lambda$  is the wavelength of the sinusoidal excitation applied to each array element. The expression in (15) is readily verified by employing Euler's formula and carrying out the binomial expansion. When  $n$  is an odd integer, the resulting exponential terms correspond to the  $n$  elements placed at  $x=0, \pm d/2, \dots, \pm (n-1)d/2$ . When  $n$  is an even integer, the resulting exponential terms correspond to the  $n$  elements placed at  $x = \pm d/2, \pm 3d/2, \dots, (n-1)d/2$ .

As before, assume the constraint in (9) is satisfied. Because of (15), the properties previously noted for the 2-element interferometer apply to the  $n$ -element interferometer. However, the first  $(n-2)$  derivatives of the array factor with respect to  $\alpha$  are now zero at each null. As a consequence, both the broad and narrow nulls have their angular widths significantly widened.

For example, when  $k_s = 3$ , the four narrow nulls have  $-30$  dB widths of approximately  $0.8^\circ$  for  $n=2$ ,  $4.4^\circ$  for  $n=3$ ,  $8^\circ$  for  $n=4$  and  $11^\circ$  for  $n=5$  while the two broad nulls in the end-fire positions at  $\alpha = \pm 90^\circ$  have  $-30$  dB widths of approximately  $12.8^\circ$  for  $n=2$ ,  $31^\circ$  for  $n=3$ ,  $42^\circ$  for  $n=4$  and  $50^\circ$  for  $n=5$ .

Because of their broader angular widths, the narrow nulls of the  $n$ -element interferometer can now be used to achieve spatial orthogonality between the host radar and interferometer. In addition, these nulls can be steered by introducing a phase shift of  $\delta$  between successive interferometer elements. The steered array factor in the  $x$ - $y$  plane is given by

$$F(n; \frac{\pi}{2}, \alpha, \delta) = \left\{ \cos \left[ k_s \frac{\pi}{2} \sin \alpha + \frac{\delta}{2} \right] \right\}^{n-1} \quad (16)$$

where  $\theta = \frac{\pi}{2}$  and (9) is invoked.

#### IV. HOST RADAR WAVEFORM

Frequency diversity is proposed for the host radar waveform. In particular, the radar pulse is assumed to consist of  $N$  contiguous frequency hops, each of duration,  $T$ . The hopping frequencies are selected from the set of equally spaced frequencies given by

$$f_n = f_c + \frac{n}{T} \quad (17)$$

where  $n = 0, 1, \dots, (N-1)$ . Frequency hopping is a flexible scheme in the sense that a discrete frequency approximation can be obtained for any frequency modulated radar pulse by appropriately ordering the frequency hops. For example, an approximation to a linear FM chirp results by simply selecting the hop frequencies to be consecutive.

Alternatively, the frequencies can be scrambled. Costas [1] showed that the frequencies from (17) can be ordered to yield a waveform whose delay-Doppler ambiguity function approximates that of the ideal impulse function (i.e., a thumb-shaped ambiguity function with a relatively low pedestal). Furthermore, for a specified value of  $N$ , there are many different orderings of the frequencies that result in essentially the same thumb shaped ambiguity function. Coherent reference denial is

enhanced by changing the frequency scrambling from one radar pulse to the next. This prevents the non-cooperative bistatic radar from relying on the integration of a single radar waveform estimate over the coherent processing interval. Instead the non-cooperative radar is forced to repeatedly estimate a new radar waveform from each pulse received in the direct path.

Another benefit from modifying the frequency scrambling from one radar pulse to the next is that the host radar is likely to be less susceptible to interfering FM signals and repeater jammers.

Also, since the radar pulse consists of  $N$  different frequency bursts, the receiver can be channelized with  $N$  channels where each channel is tuned to a unique frequency and has a bandwidth equal to  $\Delta f = 1/T$ . Channelization allows for selective limiting in each channel. In a conventional FM receiver using a single limiter, strong CW interference anywhere in the total signal bandwidth is likely to capture the receiver. As a result, the target return signals are suppressed and unlikely to be detected with a preset threshold crossing. In a channelized system, CW interference suppresses target returns in only one of the channels. Hence, the target signal power is reduced only by a factor of  $[(N-1)/N]^2$ . If  $N > 10$ , CW interference can be 20 to 30 dB above the per-channel target signal power without preventing target detection in a fixed-threshold system.

With the above considerations in mind, the host radar signal is chosen to be a frequency-hopped pulse of duration  $NT$  whose frequencies are equally spaced by  $1/T$ .

#### V. INTERFEROMETER WAVEFORM

Having selected the radar waveform, orthogonal frequency division multiplexing (OFDM) becomes an attractive technique to implement in the interferometer so as to both mask the host radar signal and communicate effectively [2]. An OFDM signal consists of  $K$  subcarriers which are on simultaneously and whose frequencies are equally spaced by  $1/T_s$ , where  $T_s$  is the modulation symbol duration. By selecting  $K=N$  and  $T_s=T$ , the frequencies of the host radar and OFDM waveforms can be made identical. In addition, the use of  $M$ -ary amplitude and phase shift keying to modulate the subcarriers makes it very difficult to separate the host radar and OFDM signals when their amplitude transitions are time synchronized.

Utilization of OFDM for communications has many important advantages. Because frequency-selective and time-variant communications channels are typically encountered at radar frequencies, channel equalization must be incorporated to achieve an acceptable level of performance. For high data-rate applications OFDM, in effect, divides the radio channel into many narrowband subchannels which appear to be frequency nonselective and time invariant. Thus, the task of channel equalization is simplified to the estimation of a single complex channel transfer function for each subcarrier. In addition, both modulation and demodulation can be implemented using fast Fourier transforms and with or without differential encoding. This simplifies transmitter and receiver implementations. Finally, the parallel nature of OFDM makes it an efficient scheme for data transmission.

## VI. NON-COOPERATIVE RADAR PERFORMANCE

As pointed out in (7), the non-cooperative bistatic radar detection statistic is obtained by correlating the received signal with an estimate of the host radar signal. In this paper it is assumed that the estimate consists of the direct path signal observed by the bistatic radar. It follows that the detection statistic is a sum of terms involving the delay-Doppler ambiguity functions of the host radar and interferometer waveforms, their cross ambiguity function and additive noise.

Assuming Gaussian additive noise, the detection statistic in the absence of a target is a zero-mean complex Gaussian random variable with variance,  $\sigma_{\ell_0}^2$ . The false alarm probability of the non-cooperative radar is given by

$$P_F = \exp\left[-\gamma^2 / 2\sigma_{\ell_0}^2\right] \quad (18)$$

where  $\gamma$  is the threshold.

In the presence of a target, the detection statistic is a random variable with non-zero mean,  $m_{\ell_1}$ , and variance,  $\sigma_{\ell_1}^2$ . Taking advantage of the result that this statistic is a sum of many random terms, none of which are dominant, the statistic is approximated as a complex Gaussian random variable. The

detection probability of the non-cooperative radar is given by

$$P_D = 1 - \frac{1}{\sqrt{2} \sigma_{\ell_1}} T_\gamma \left[ 1, 0, \frac{|m_{\ell_1}|}{\sqrt{2} \sigma_{\ell_1}} \right] \quad (19)$$

where  $T_\gamma(m, n, p)$  is the Toronto function [3].

## VII. FUTURE WORK

Future work consists of 1) development of joint spatial-temporal antenna-based signal processing techniques and waveforms that can be applied simultaneously to radar and communication systems without self jamming, 2) design of multi-dimensional waveforms for bistatic radar denial and embedded communications, 3) design of diverse waveforms which will accommodate multi-mission operations such as ground and airborne moving target indication, tracking, automatic target recognition and foliage and ground penetration, 4) interference reduction techniques by spectrum partitioning and 5) generalization of the ambiguity function to include direction of arrival.

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